

Collective excitations across the BCS-BEC crossover induced by a synthetic Rashba spin-orbit coupling

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Synthetic non-Abelian gauge fields in cold atom systems produce a generalized Rashba spin-orbit interaction described by a vector $\lambda = (\lambda_x, \lambda_y, \lambda_z)$ that influences the motion of spin- $\frac{1}{2}$ fermions. It was recently shown [Phys. Rev. B 84, 014512 (2011)] that on increasing the strength of the spin-orbit coupling $\lambda = |\lambda|$, a system of fermions at a finite density $\rho \approx k_F^3$ evolves to a BEC like state even in the presence of a weak attractive interaction (described by a scattering length a_s). The BEC obtained at large spin-orbit coupling ($\lambda \gg k_F$) is a condensate of rashbons – novel bosonic bound pairs of fermions whose properties are determined solely by the gauge field. In this paper, we investigate the collective excitations of such superfluids by constructing a Gaussian theory using functional integral methods. We derive explicit expressions for superfluid phase stiffness, sound speed and mass of the Anderson-Higgs boson that are valid for any λ and scattering length. We find that at finite λ , the phase stiffness is always lower than that set by the density of particles, consistent with earlier work [arXiv:1110.3565] which attributed this to the lack of Galilean invariance of the system at finite λ . We show that there is an *emergent Galilean invariance* at large λ , and the phase stiffness is determined by the rashbon density and mass, consistent with Leggett's theorem. We further demonstrate that the rashbon BEC state is a superfluid of anisotropic rashbons interacting via a contact interaction characterized by a rashbon-rashbon scattering length a_R . We show that a_R goes as λ^{-1} and is essentially *independent* of the scattering length between the fermions as long as it is nonzero. Analytical results are presented for a rashbon BEC obtained in a spherical gauge field with $\lambda_x = \lambda_y = \lambda_z = \frac{\lambda}{\sqrt{3}}$.

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I. INTRODUCTION

The simulation of quantum condensed matter systems¹⁻³ with cold atoms has captivated the imagination and efforts of many. Some of the most recent new developments include the generation⁴⁻⁸ of synthetic gauge fields in bosons⁹⁻¹¹ and realization of fermionic degeneracy in their presence.¹²

Uniform non-Abelian gauge fields produce spin-orbit interactions. The physics of bosons in spin-orbit coupled system has been investigated by many authors.¹³⁻¹⁵ The rich physics hidden in the fermion problem was revealed by the solution of the two-body problem given in ref. [16], where it was shown that for certain high symmetry gauge fields, a bound state appears *even for an infinitesimal* attraction in the singlet channel. The key outcome of this is that a BCS-BEC crossover is induced by increasing the strength of the gauge field even with a weak attractive interaction.¹⁷ The BEC that is realized was shown to be a condensate of a new type of boson – the rashbon – whose properties are determined solely by the gauge field and not by the scattering length characterizing the interaction between the fermions. This BEC realized at large gauge coupling is called the rashbon-BEC (RBEC). Concurrently, anisotropic superfluidity of rashbons¹⁸, zero-temperature BCS-BEC crossover in the presence of Zeeman fields^{19,20} (imbalance) was studied, and transition temperatures were estimated^{21,22}. Dresselhaus like spin-orbit interaction^{23,24} has also been ex-

amined. Non-Abelian gauge fields in lower dimensions and lattices have also been investigated.²⁵⁻²⁷ A review of these fast paced recent developments may be found in ref. [28]. Several aspects of the physics of spin-orbit coupled fermions were reported earlier^{29,30} and were independently discovered in the cold atoms context.^{16,17}

The motivating questions for this work pertain to the properties of the RBEC that is obtained at large gauge coupling at a fixed scattering length a_s . In the usual BCS-BEC crossover³¹⁻³⁵ in the absence of spin-orbit interaction, the BEC state for small positive scattering length a_s is a condensate of bosons (fermionic dimer molecules). This BEC state can be described by the Bogoliubov theory of interacting bosons³⁶, where the boson mass is twice the fermion mass and the effective boson-boson scattering length is proportional to a_s .^{35,37} Does a similar description hold for the RBEC obtained by tuning the magnitude of the gauge coupling? How does rashbon-rashbon scattering enter the description, i. e., what is the effective rashbon-rashbon scattering length?

That collective excitations have interesting and unusual features was pointed out in ref. [38] which studied phase stiffness K^s (superfluid density) for an extreme-oblate gauge field (see below for a definition). In the regime $\lambda \lesssim k_F$, the K^s decreases with increasing gauge coupling. However, for $\lambda \gtrsim k_F$, K^s increases and saturates as λ/k_F attains large values. For all λ , K^s is less than $\rho/4m$, the value of phase stiffness for a superfluid without the spin-orbit interaction, where ρ the density and m is the mass of the fermions. This is attributed³⁸ to

the lack of Galilean invariance in systems with synthetic non-Abelian gauge fields (see also, ref. [39]). While this is true, we conjecture that Galilean will be approximately restored in the system for $\lambda \gg k_F$ when an attractive interaction, however weak, is present. The basis of this conjecture stems from the fact that at large λ the system with even a weak attraction can be thought of as a collection of rashbons which disperse quadratically²², $\varepsilon_R(\mathbf{q}) = -E^R + \sum_i \frac{q_i^2}{2m_i^R}$, albeit with an anisotropic dispersion defined by the direction dependent rashbon mass m_i^R and E^R is the rashbon binding energy, a result that is valid for $|\mathbf{q}| \ll \lambda$. This dispersion is Galilean invariant, and therefore we expect to obtain a phase stiffness tensor $K_{ij}^s = \frac{\rho_R}{m_i^R} \delta_{ij}$ (no sum on i), where $\rho_R = \rho/2$ is the rashbon density, consistent with Leggett's result^{33,40}. Testing this conjecture regarding *emergent Galilean invariance* and answering the questions raised in the previous paragraph are the aims of this paper.

To this end, we investigate the collective excitations of superfluids induced by non-Abelian gauge fields using a Gaussian fluctuations theory with a functional integral framework. Our main result is that the rashbon BEC can be described as a collection of weakly interacting rashbons. We obtain an effective rashbon-rashbon scattering length which we show is generically proportional to λ^{-1} , and is *independent of the scattering length between the fermions* to leading order. In addition, we show that the phase stiffness has precisely the form as conjectured above. The RBEC state is a remarkable state where the effective interaction between the emergent bosons (rashbons) is determined by the *kinetic energy* (spin-orbit coupling λ) of the constituent fermions, and *not* the attraction between the fermions as long as it is non-vanishing. Our theory also provides the phase stiffness, speed of sound and the mass of the Anderson-Higgs boson for any gauge coupling.

Sec. II outlines the functional integral framework used in the analysis of the collective excitations and obtains general formulae for the phase stiffness, sound speed and Anderson-Higgs mass for a generic Rashba like spin-orbit coupled system. Results for a spherical gauge field are discussed in sec. III, and sec. IV contains a discussion of the properties of rashbon BECs. The paper is summarized in sec. V.

II. FORMULATION

We follow closely the notation and terminology introduced in^{16,17}. The Hamiltonian of the system of interest is made up of two pieces

$$\mathcal{H} = \mathcal{H}_R + \mathcal{H}_v. \quad (1)$$

The kinetic energy of the spin- $\frac{1}{2}$ fermions is

$$\mathcal{H}_R = \sum_{\mathbf{k}} \varepsilon_{\alpha}(\mathbf{k}) C_{\mathbf{k}\alpha}^{\dagger} C_{\mathbf{k}\alpha} \quad (2)$$

where, C s and C^{\dagger} s are fermion operators,

$$\varepsilon_{\alpha}(\mathbf{k}) = \frac{k^2}{2} - \alpha |\mathbf{k}_{\lambda}|, \quad (3)$$

$\alpha = \pm 1$ is the helicity, $\mathbf{k}_{\lambda} = \lambda_x k_x \mathbf{e}_x + \lambda_y k_y \mathbf{e}_y + \lambda_z k_z \mathbf{e}_z$. The “vector” $\boldsymbol{\lambda} \equiv (\lambda_x, \lambda_y, \lambda_z) \equiv \lambda \hat{\boldsymbol{\lambda}}$ describes the configuration of the gauge field that induces a generalized Rashba spin-orbit interaction, where $\lambda = |\boldsymbol{\lambda}|$ is the magnitude of the gauge coupling and $\hat{\boldsymbol{\lambda}}$ is a unit vector. High symmetry gauge field configurations of interest include the extreme oblate (EO) gauge field with $\boldsymbol{\lambda} = \lambda \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)$ and the spherical (S) gauge field which has $\boldsymbol{\lambda} = \lambda \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$. We use units where the fermion mass m and \hbar are unity. We consider a finite density of fermions ρ which defines a momentum scale k_F such that $\rho = \frac{k_F^3}{3\pi^2}$, and an energy scale $E_F = \frac{k_F^2}{2}$.

The interaction piece \mathcal{H}_v describes an attraction in the singlet channel as

$$\mathcal{H}_v = \frac{v}{\Omega} \sum_{\mathbf{q}, \mathbf{k}, \mathbf{k}'} C_{(\frac{\mathbf{q}}{2} + \mathbf{k})\uparrow}^{\dagger} C_{(-\frac{\mathbf{q}}{2} + \mathbf{k}')\downarrow}^{\dagger} C_{\mathbf{k}'\downarrow} C_{\mathbf{k}\uparrow} \quad (4)$$

where Ω is the volume of the system, v is the bare interaction parameter. The theory requires an ultraviolet cutoff Λ which can be eliminated by using $\frac{1}{4\pi a_s} = \frac{1}{v} + \Lambda$. Using mean-field theory, it was shown in ref. [17] that increasing λ induces a BCS to BEC crossover even for a weak attractive interaction ($|k_F a_s| \ll 1, a_s < 0$). We aim to study the collective excitations of such superfluids across this crossover.

To this end we use a functional integral framework which has been extensively used in the study of BCS-BEC crossover.^{35,41–45} Denoting inverse temperature as β and chemical potential as μ , we write the action

$$\mathcal{S}[\Psi] = \sum_{\mathbf{k}} \Psi^{\star}(\mathbf{k}) (-G_0^{-1}(\mathbf{k}, \mathbf{k}')) \Psi(\mathbf{k}') + \frac{v}{\beta\Omega} \sum_{\mathbf{q}} S^{\star}(\mathbf{q}) S(\mathbf{q}) \quad (5)$$

where

$$\Psi(\mathbf{k}) = \begin{pmatrix} c_{+}(\mathbf{k}) \\ c_{+}^{\star}(-\mathbf{k}) \\ c_{-}(\mathbf{k}) \\ c_{-}^{\star}(-\mathbf{k}) \end{pmatrix} \quad (6)$$

is a Nambu vector consisting of Grassmann variables describing the fermions, $\mathbf{k} = (ik_n, \mathbf{k})$ where ik_n is a fermionic Matsubara frequency,

$$G_0^{-1}(\mathbf{k}, \mathbf{k}') = \begin{pmatrix} ik_n - \xi_{+}(\mathbf{k}) & 0 & 0 & 0 \\ 0 & ik_n + \xi_{+}(\mathbf{k}) & 0 & 0 \\ 0 & 0 & ik_n - \xi_{-}(\mathbf{k}) & 0 \\ 0 & 0 & 0 & ik_n + \xi_{-}(\mathbf{k}) \end{pmatrix} \delta_{\mathbf{k}, \mathbf{k}'}, \quad (7)$$

$\xi_{\alpha}(\mathbf{k}) = \varepsilon_{\alpha}(\mathbf{k}) - \mu$, and

$$S^{\star}(\mathbf{q}) = \sum_{\mathbf{k}, \alpha\beta} A_{\alpha\beta}(\mathbf{q}, \mathbf{k}) c_{\alpha}^{\star}(\frac{\mathbf{q}}{2} + \mathbf{k}) c_{\beta}^{\star}(\frac{\mathbf{q}}{2} - \mathbf{k}) \quad (8)$$

is the Fourier transform of the singlet density with $q = (iq_\ell, \mathbf{q})$, iq_ℓ is a bosonic Matsubara frequency. $A_{\alpha\beta}(\mathbf{q}, \mathbf{k})$ is the singlet amplitude in a two particle state of α and β helicities, with centre of mass momentum \mathbf{q} and relative momentum \mathbf{k} . It must be noted that $A_{\alpha\beta}(\mathbf{q}, \mathbf{k})$ satisfy many symmetry properties which are used extensively in the work that follows. Moreover, care must be exercised in the definition of $A_{\alpha\beta}(\mathbf{q}, \mathbf{k})$ due to the non-zero Chern flux originating from the origin of the momentum space (see ref. [46]).

We now introduce a Hubbard-Stratanovich pair field $\Delta(q)$ to decouple the interaction term to obtain

$$\mathcal{S}[\Psi, \Delta] = \sum_{k, k'} \Psi^*(k) (-G^{-1}(k, k')) \Psi(k') - \frac{1}{v} \sum_q \Delta^*(q) \Delta(q) \quad (9)$$

where $G^{-1}(k, k')$ is

$$G^{-1}(k, k') = G_0(k, k') - \Delta(k, k'), \quad (10)$$

$$\Delta(k, k') = \begin{pmatrix} 0 & \Delta_{++}(k, k') & 0 & \Delta_{+-}(k, k') \\ \tilde{\Delta}_{++}(k, k') & 0 & \tilde{\Delta}_{+-}(k, k') & 0 \\ 0 & \Delta_{-+}(k, k') & 0 & \Delta_{--}(k, k') \\ \tilde{\Delta}_{-+}(k, k') & 0 & \tilde{\Delta}_{--}(k, k') & 0 \end{pmatrix} \quad (11)$$

with

$$\Delta_{\alpha\beta}(k, k') = \sum_q \frac{\Delta(q)}{\sqrt{\beta\Omega}} A_{\alpha\beta}(\mathbf{q}, \mathbf{k} - \frac{\mathbf{q}}{2}) \delta_{q, k-k'} \quad (12)$$

$$\tilde{\Delta}_{\alpha\beta}(k, k') = \sum_q \frac{\Delta^*(-q)}{\sqrt{\beta\Omega}} A_{\beta\alpha}(-\mathbf{q}, \mathbf{k} - \frac{\mathbf{q}}{2}) \delta_{q, k-k'} \quad (13)$$

We integrate out the fermions to obtain the action only in terms of the pairing field

$$\mathcal{S}[\Delta] = -\frac{1}{v} \sum_q \Delta^*(q) \Delta(q) - \ln \det[-G] \quad (14)$$

We now perform a saddle point analysis of the action and look for static and homogeneous solutions via the ansatz

$$\Delta^{\text{SP}}(q) = \sqrt{\beta\Omega} \sqrt{2} \Delta_0 \delta_{q,0} \quad (15)$$

where the factor of $\sqrt{2}$ is introduced for convenience.

With this ansatz for the saddle point, the Green's function $G(k, k')$ is

$$G(k, k') = \begin{pmatrix} G_+^p(k) & G_+^a(k) & 0 & 0 \\ -G_+^a(k) & G_+^h(k) & 0 & 0 \\ 0 & 0 & G_-^p(k) & G_-^a(k) \\ 0 & 0 & -G_-^a(k) & G_-^h(k) \end{pmatrix} \delta_{k, k'} \quad (16)$$

where

$$G_\alpha^p(k) = \frac{ik_n + \xi_\alpha(\mathbf{k})}{(ik_n)^2 - E_\alpha^2(\mathbf{k})} \quad (17)$$

$$G_\alpha^h(k) = \frac{ik_n - \xi_\alpha(\mathbf{k})}{(ik_n)^2 - E_\alpha^2(\mathbf{k})} \quad (18)$$

$$G_\alpha^a(k) = \frac{i\alpha\Delta_0}{(ik_n)^2 - E_\alpha^2(\mathbf{k})} \quad (19)$$

with $E_\alpha(\mathbf{k}) = \sqrt{\xi_\alpha(\mathbf{k})^2 + \Delta_0^2}$. The saddle point condition, after appropriate frequency sums, is

$$-\frac{1}{v} = \frac{1}{2\Omega} \sum_{\mathbf{k}\alpha} \frac{\tanh \frac{\beta E_\alpha(\mathbf{k})}{2}}{2E_\alpha(\mathbf{k})} \quad (20)$$

and agrees with the gap equation derived in ref. [17 and 22]. The saddle point number equation is

$$\rho = \frac{1}{2\Omega} \sum_{\mathbf{k}\alpha} \left(1 - \frac{\xi_\alpha(\mathbf{k})}{E_\alpha(\mathbf{k})} \right) \quad (21)$$

The values of Δ_0 and μ are set by the simultaneous solution of eqn. (20) and eqn. (21).

Collective excitations of the system are described by fluctuations about the saddle point state. We treat them at Gaussian level by introducing “small oscillations” about the saddle point value of the pairing field,

$$\Delta(q) = \Delta^{\text{SP}}(q) + \eta(q) \quad (22)$$

After some straightforward, if lengthy, algebra, the action to quadratic order in η is

$$\mathcal{S}[\eta] = \mathcal{S}^{\text{SP}} + \frac{1}{2} \sum_q \begin{pmatrix} \eta^*(q) & \eta(-q) \end{pmatrix} \mathbf{\Pi}(q) \begin{pmatrix} \eta(q) \\ \eta^*(-q) \end{pmatrix} \quad (23)$$

where

$$\begin{aligned} \mathbf{\Pi}(q) &= \begin{pmatrix} \Pi_{11}(q) & \Pi_{12}(q) \\ \Pi_{21}(q) & \Pi_{22}(q) \end{pmatrix} \\ \Pi_{11}(q) &= \Pi_{22}(-q) = -\frac{1}{v} + \frac{1}{\beta\Omega} \sum_{\mathbf{k}, \alpha\beta} |A_{\alpha\beta}(\mathbf{q}, \mathbf{k})|^2 G_\alpha^p(iq_\ell + ik_n, \frac{\mathbf{q}}{2} + \mathbf{k}) G_\beta^h(ik_n, -\frac{\mathbf{q}}{2} + \mathbf{k}) \\ \Pi_{12}(q) &= \Pi_{21}(q) = -\frac{1}{\beta\Omega} \sum_{\mathbf{k}, \alpha\beta} \alpha\beta |A_{\alpha\beta}(\mathbf{q}, \mathbf{k})|^2 G_\alpha^a(iq_\ell + ik_n, \frac{\mathbf{q}}{2} + \mathbf{k}) G_\beta^a(ik_n, -\frac{\mathbf{q}}{2} + \mathbf{k}) = \Pi_{12}(-q) = \Pi_{21}(-q) \end{aligned} \quad (24)$$

Collective excitations of a superfluid can be conveniently described in terms of spatio-temporally dependent phase and amplitude oscillations. We, therefore, express η in terms of two other real fields ζ (amplitude

oscillation) and ϕ (phase).

fluctuation) and ϕ (phase fluctuation) as

$$\eta(q) = \Delta_0 (\zeta(q) + i\phi(q)) \quad (25)$$

with $\zeta(-q) = \zeta^*(q)$ and $\phi(-q) = \phi^*(q)$. The action in terms of these two fields is

$$\mathcal{S}[\zeta, \phi] = \mathcal{S}^{sp} + \frac{1}{2} \sum_q \begin{pmatrix} \zeta^*(q) & \phi^*(q) \end{pmatrix} \Gamma(q) \begin{pmatrix} \zeta(q) \\ \phi(q) \end{pmatrix} \quad (26)$$

where, using eqn. (24), we find

$$\Gamma(q) = \begin{pmatrix} \Gamma_{\zeta\zeta}(q) & \Gamma_{\zeta\phi}(q) \\ \Gamma_{\phi\zeta}(q) & \Gamma_{\phi\phi}(q) \end{pmatrix} \quad (27)$$

$$\Gamma_{\zeta\zeta}(q) = \Delta_0^2 (\Pi_{11}(q) + \Pi_{11}(-q) + 2\Pi_{12}(q)) \quad (28)$$

$$\Gamma_{\zeta\phi}(q) = i\Delta_0^2 (\Pi_{11}(q) - \Pi_{11}(-q)) = -\Gamma_{\phi\zeta}(q) \quad (29)$$

$$\Gamma_{\phi\phi}(q) = \Delta_0^2 (\Pi_{11}(q) + \Pi_{11}(-q) - 2\Pi_{12}(q)) \quad (30)$$

We now perform the necessary frequency sums to obtain expressions for the Γ s. Here and henceforth in this paper, we focus at zero temperature ($T = 0$) and “small” q , and do not show the lengthy expressions valid for any temperature and q . For small q at $T = 0$, we have,

$$\Gamma_{\phi\phi}(iq_\ell, \mathbf{q}) = q_i K_{ij}^s q_j - Z(iq_\ell)^2 \quad (31)$$

$$\Gamma_{\zeta\phi}(iq_\ell, \mathbf{q}) = -iq_\ell X \quad (32)$$

$$\Gamma_{\zeta\zeta}(iq_\ell, \mathbf{q}) = U + q_i V_{ij} q_j - W(iq_\ell)^2 \quad (33)$$

where the quantities K^s, Z, X, U, V, W depend on the saddle point values of Δ_0 and μ . K_{ij}^s is the phase stiffness given by

$$K_{ij}^s = \frac{\Delta_0^2}{2\Omega} \sum_{\mathbf{k}\alpha} \frac{v_i^\alpha(\mathbf{k}) v_j^\alpha(\mathbf{k})}{4E_\alpha^3(\mathbf{k})} + \frac{2\Delta_0^2}{\Omega} \sum_{\mathbf{k}} \frac{(\varepsilon_+(\mathbf{k}) - \varepsilon_-(\mathbf{k}))^2}{2E_+(\mathbf{k})E_-(\mathbf{k})(E_+(\mathbf{k}) + E_-(\mathbf{k}))} S_{ij}(\mathbf{k}) \quad (34)$$

where $v_i^\alpha(\mathbf{k}) = \frac{\partial \varepsilon_\alpha(\mathbf{k})}{\partial k_i}$, and $S_{ij}(\mathbf{k})$ is a tensor that defines the singlet amplitude $A_{+-}(\mathbf{q}, \mathbf{k})$ for small \mathbf{q} as

$$|A_{+-}(\mathbf{q}, \mathbf{k})|^2 = |A_{-+}(\mathbf{q}, \mathbf{k})|^2 \approx q_i S_{ij}(\mathbf{k}) q_j. \quad (35)$$

It must be noted that extensive use of the properties of $A_{\alpha\beta}(\mathbf{q}, \mathbf{k})$ is made in arriving at this expression for the phase stiffness tensor that is valid for *any* gauge field. The other quantities in eqn. (31),

$$\begin{aligned} Z &= \frac{\Delta_0^2}{2\Omega} \sum_{\mathbf{k}\alpha} \frac{1}{4E_\alpha^3(\mathbf{k})} \\ X &= \frac{\Delta_0^2}{2\Omega} \sum_{\mathbf{k}\alpha} \frac{\xi_\alpha(\mathbf{k})}{2E_\alpha^3(\mathbf{k})} \\ U &= \frac{\Delta_0^4}{2\Omega} \sum_{\mathbf{k}\alpha} \frac{1}{E_\alpha^3(\mathbf{k})} \\ W &= Z - \frac{\Delta_0^4}{2\Omega} \sum_{\mathbf{k}\alpha} \frac{1}{4E_\alpha^5(\mathbf{k})}. \end{aligned} \quad (36)$$

We have not shown the expression for V_{ij} since it will not be used in the discussion below.

The dispersion of the excitations can be obtained by first analytically continuing $iq_\ell \rightarrow \omega^+$ to real frequencies and solving $\det \Gamma(\omega^+, \mathbf{q}) = 0$. We obtain two modes for a given $\mathbf{q} = q\hat{\mathbf{q}}$, one is a *gapless* sound mode and other is the *gapped* Anderson-Higgs mode. The speed of sound along direction $\hat{\mathbf{q}}$ is given by

$$c_s^2(\hat{\mathbf{q}}) = \frac{\hat{q}_i K_{ij}^s \hat{q}_j}{Z + \frac{X^2}{U}} \quad (37)$$

and the mass of the Anderson-Higgs mode M_{AH} is obtained as

$$M_{AH}^2 = \frac{ZU + X^2}{ZW} \quad (38)$$

It must be noted that the amplitude and phase modes are coupled⁴²; their coupling is determined by the quantity X .

Equations 34, 37 and 38 are the key results of this paper for the collective excitations of spin-orbit coupled superfluids that are applicable to *any* Rashba gauge field and scattering length at zero temperature. We have not shown the finite temperature results here to avoid lengthy expressions. In the remainder of the paper, we illustrate the physics of these formulae using the spherical gauge field (next section) and explore the consequences of our results particularly for the rashbon-BEC (sec. IV).

III. COLLECTIVE EXCITATIONS FOR THE SPHERICAL GAUGE FIELD

In this section we discuss collective excitations of superfluids realized in a spherical gauge field with $\boldsymbol{\lambda} = \lambda \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$ as noted earlier. The two body problem for this gauge field was exhaustively investigated in ref. [16] where an analytical expression for the binding energy valid for *any scattering length* is derived along with an analytical expression for the bound state wave function. The binding energy of the rashbon¹⁶ is

$$E^R = \frac{\lambda^2}{3} \quad (39)$$

and the rashbon mass (in units of fermion mass) is²²

$$m^R = \frac{3}{7}(4 + \sqrt{2}) \quad (40)$$

A route to experimental realization of this gauge field has recently been suggested.⁴⁷ A detailed study of two-body scattering from a finite range box potential is carried out in ref. [48].

A. Analytical Results

Analytical results can be obtained in two regimes of λ . These correspond to $\lambda \ll k_F$, and the other to $\lambda \gg \max(k_F, 1/a_s)$.

1. $\lambda \ll k_F$

Two regimes of a_s are tractable analytically for this regime of λ , both of which are well known; we state them here for the sake of completion.

I. $a_s < 0, |k_F a_s| \ll 1$: This regime is studied in detail in ref. [17]. The chemical potential in this regime is set by the value of the noninteracting system (which falls by an amount proportional to $\frac{\lambda^2}{k_F^2}$). The gap Δ_0 is essentially unaltered from the well known BCS value. Under these conditions, we obtain the phase stiffness to be $\frac{\rho}{4}$ with a fall of order $\frac{\lambda^2}{k_F^2}$. The leading term in the speed of sound is $k_F/\sqrt{3}$ as shown by Anderson⁴⁹ (with a fall proportional to λ^2/k_F^2) and the Anderson-Higgs mass is exponentially small. This limit corresponds essentially to the BCS limit studied in ref. [42].

II. $a_s > 0, k_F a_s \ll 1$: This corresponds to the usual BEC regime (ref. [37 and 42]). Here the chemical potential $\mu = -\frac{1}{2a_s^2} + 2\pi a_s \rho$ and the gap $\Delta_0^2 = \frac{4\pi\rho}{a_s}$. The phase stiffness $K^s = \frac{\rho}{4}$, speed of sound is $c_s^2 = 2\pi\rho a_s$, the $M_{AH} = \frac{4}{a_s^2}$. In this regime, the amplitude and the phase modes are strongly mixed.

2. $\lambda \gg k_F$ and $\lambda \gg \frac{1}{a_s}$

This is the regime of interest and corresponds to the rashbon BEC. In this regime, we report new results for the gap

$$\Delta_0^2 = \frac{2\pi}{\rho} \frac{\lambda}{\sqrt{3}} \quad (41)$$

and the chemical potential

$$\mu = -\frac{E^R}{2} + \pi\rho \frac{\sqrt{3}}{\lambda}. \quad (42)$$

By an analysis of the expression for the phase stiffness (eqn. (34)) which is isotropic for this gauge field, we find that

$$K^s = \frac{\rho}{2m^R} \quad (43)$$

precisely as conjectured in the introductory section (see below for further discussion). Additional analysis provides

$$c_s^2 = \frac{2\pi\rho}{m^R} \left(\frac{\sqrt{3}}{\lambda} \right) \quad (44)$$

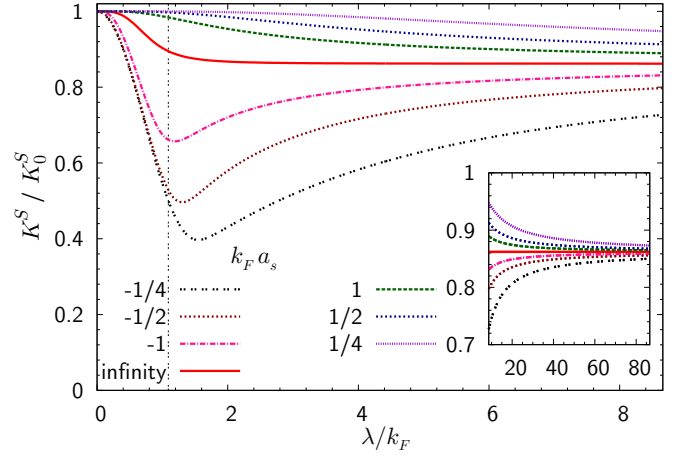


FIG. 1. (color online) **Phase stiffness** - Evolution of phase stiffness K^s with increasing λ for the spherical gauge field for various scattering lengths. $K_0^s = \rho/4$. The inset shows that K^s/K_0^s tends to $2/m^R$ for large λ demonstrating the emergent Galilean invariance. The dashed vertical line corresponds to $\lambda = \lambda_T$ where there is a change in the topology of the Fermi surface of the non-interacting system.¹⁷

and

$$M_{AH} = \frac{2}{3}\lambda^2. \quad (45)$$

As expected, the leading terms for all the quantities of interest are *independent of the scattering length between the fermions*; scattering length corrections (which we do not show) appear as powers of $(1/\lambda a_s)$, which in this regime are small. We emphasize that in this RBEC regime the amplitude and the phase mode are strongly coupled, just like in the usual BEC regime.

B. Numerical Results

In this section we show the results of numerical calculations of evolution of K^s , c_s and M_{AH} with increasing λ for several scattering lengths.

1. Superfluid Phase Stiffness

Fig. 1 shows a plot of the phase stiffness as a function of λ for various scattering lengths. We see that for small negative scattering lengths, the behaviour of K^s is non-monotonic; it decreases with increasing λ and attains a minimum near $\lambda \gtrsim \lambda_T$. This is fully consistent with the finding of ref. [38] for the EO gauge field. The new aspect uncovered in our work is that for $\lambda \gg \max(k_F, 1/a_s)$, the phase stiffness tends to that of a *collection of interacting rashbons* in exactly same way as the motivating conjecture of this paper. In other words, $K^s(\lambda \rightarrow \infty) = \frac{\rho_R}{m^R}$ where $\rho_R = \rho/2$ is the rashbon number density. The physics behind this is that the rashbon

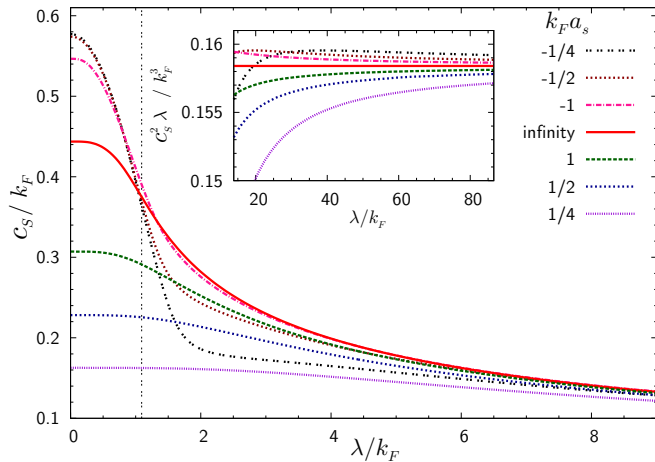


FIG. 2. (color online) **Sound speed** - Evolution of the sound speed c_s with increasing λ for the spherical gauge field for various scattering lengths. The inset shows that c_s^2 has the behaviour obtained in eqn. (44), independent of the scattering length. The dashed vertical line corresponds to $\lambda = \lambda_T$ where there is a change in the topology of the Fermi surface of the non-interacting system.

dispersion $\varepsilon_R(\mathbf{q}) = -E^R + \frac{q^2}{2m^R}$ is Galilean invariant, and hence the phase stiffness as found at $\lambda \rightarrow \infty$ is consistent with Leggett's result^{33,40}. This is a remarkable feature, and corresponds to an *emergent infrared symmetry*, i. e., *in the presence of interactions however small, the system organizes itself to posses a larger symmetry at low energies!* A important point that can be inferred is that the nonzero phase stiffness implies that rashbons are *interacting* bosons. The nature of the interaction is uncovered in the next section.

2. Sound Speed

The variation of the sound speed with increasing λ is shown in fig. 2. We see that there is a monotonic decrease in the sound speed with increasing λ for all scattering lengths. At large λ , the sound speed is inversely proportional to λ as obtained analytically (see eqn. (44)). Again, that there is sound propagation in the medium suggests the presence of interactions between the rashbons.

3. Mass of the Anderson-Higgs boson

For small gauge coupling ($\lambda \ll k_F$) M_{AH} corresponds to the gap of the amplitude mode for small negative scattering lengths. This mass grows with increasing λ albeit with some features near $\lambda \sim \lambda_T$ for small negative scattering lengths. At large λ we find the expected λ^2 behaviour.

The key result of this section is that at large λ , the system behaves like a Galilean invariant collection of in-

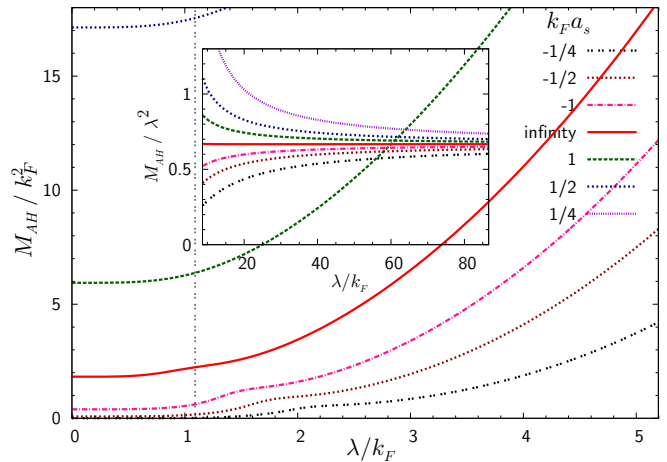


FIG. 3. (color online) **Mass of the Anderson-Higgs boson** - Evolution of mass of the Anderson-Higgs boson M_{AH} with increasing λ for the spherical gauge field for various scattering lengths is shown in Fig. 3. The inset shows that M_{AH} goes as λ^2 , independent of the scattering length (eqn. (45)). The dashed vertical line corresponds to $\lambda = \lambda_T$ where there is a change in the topology of the Fermi surface of the non-interacting system.

teracting rashbons. Since this regime is the *raison d'être* of this paper, we do not pause to consider the interesting regime of $\lambda \sim \lambda_T$ which no doubt contains rich physics.

IV. PROPERTIES OF RASHBON BOSE-EINSTEIN CONDENSATES (RBEC)

That the system evolves to a collection of interacting rashbons with increasing λ is conclusively demonstrated in the previous section. The rashbon dispersion derived in ref. [22] provides the kinetic energy of the rashbons. What about their interactions? Interestingly, the results of the previous section allow us to answer this question.

Recall from the Bogoliubov theory³⁶ that a collection of bosons of mass m_B with number density ρ_B and a contact interaction described by a scattering length a_B has a superfluid ground state at zero temperature. The chemical potential of this system is

$$\mu_B = \frac{4\pi a_B}{m_B} \rho_B \quad (46)$$

and the speed of sound is

$$c_s^B = \sqrt{\frac{\mu_B}{m_B}} = \sqrt{\frac{4\pi a_B \rho_B}{m_B^2}}. \quad (47)$$

From eqn. (42), the rashbon chemical potential μ^R (measured from the bottom of the rashbon band at $-E^R$) is

$$\mu^R = 2\pi\rho\frac{\sqrt{3}}{\lambda} \quad (48)$$

We see immediately that the speed of sound obtained in eqn. (44) is *consistent with eqn. (47) from Bogoliubov theory*

$$c_s^2 = \frac{\mu^R}{m^R} \quad (49)$$

This clearly demonstrates that the rashbon BEC is a condensate of rashbons interacting with a contact interaction. Writing

$$c_s^2 = \sqrt{\frac{4\pi a_R \rho_R}{(m^R)^2}} \quad (50)$$

allows us to calculate the rashbon-rashbon scattering length as

$$a_R = \frac{3\sqrt{3}(4 + \sqrt{2})}{7} \frac{1}{\lambda} \quad (51)$$

which is approximately equal to $\frac{4}{\lambda}$. This result is remarkable in the following sense that the effective interaction between rashbons is determined by a scale λ that enters the kinetic energy of the constituent fermions, and *not* by the interaction between the constituent fermions (scattering length a_s)!

We emphasize that although our arguments used the spherical gauge fields, the results obtained are applicable to other gauge field configurations described by a general vector $\boldsymbol{\lambda} = \lambda \hat{\boldsymbol{\lambda}}$ (except the extreme prolate gauge field which has only one nonvanishing component, see ref. [17]). For a generic gauge field, the rashbon chemical potential will be

$$\mu^R = M(\hat{\boldsymbol{\lambda}}) \frac{\rho}{\lambda} \quad (52)$$

where $M(\hat{\boldsymbol{\lambda}})$ is a dimensionless number that depends on $\hat{\boldsymbol{\lambda}}$, and the anisotropic speed of sound in the i -direction will be

$$c_s^2(i) = \frac{\mu^R}{m_i^R} \quad (53)$$

where m_i^R is the anisotropic rashbon mass²² that depends, again, on $\hat{\boldsymbol{\lambda}}$. The rashbon-rashbon scattering length will be

$$a_R = \frac{N(\hat{\boldsymbol{\lambda}})}{\lambda} \quad (54)$$

where $N(\hat{\boldsymbol{\lambda}})$ is dimensionless number determined by $\hat{\boldsymbol{\lambda}}$. The low energy properties of the rashbon BEC are similar to those of the usual Bogoliubov Bose fluid; in fact, generically, RBEC is a superfluid of anisotropically dispersing rashbons interacting with a contact potential described by a scattering that depends inversely on the spin orbit coupling strength of the fermions. It must be noted that accurate determination of $N(\hat{\boldsymbol{\lambda}})$ may require further self consistent treatment of the theory.^{45,50}

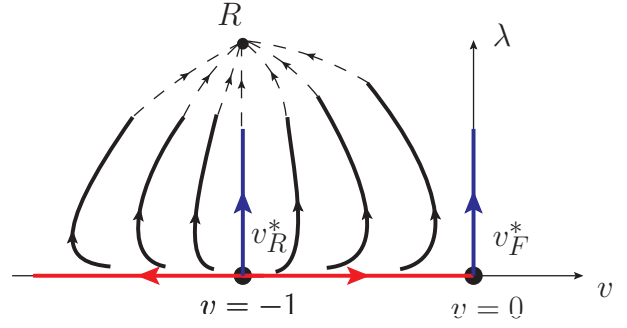


FIG. 4. (color online) **Schematic two-body RG flow diagram** - The rashbon state corresponds to the stable fixed point R at $\lambda = \infty$ and $v = -1$. Flow from any point with $\lambda \neq 0$ and $v \neq 0$ reaches R .

V. SUMMARY

In this paper, we explore the properties of the superfluids induced by non-Abelian gauge fields focusing on their collective excitations. We present results for superfluid phase stiffness, sound speed and Anderson-Higgs mass valid for *any* Rashba gauge field and scattering length. Our main results are

- Superfluid phase stiffness has non-monotonic behaviour with increasing λ , the scale of the spin-orbit interaction. This is in agreement with an earlier report³⁸ of superfluid density for the EO gauge field.
- A new result is that for large gauge coupling, i.e., in the rashbon BEC, the superfluid phase stiffness is determined by the rashbon mass²². This arises from *an emergent Galilean invariance* at infrared energies for large gauge couplings, and the phase stiffness is consistent with Leggett's result.
- The sound speed decreases monotonically with increasing gauge coupling. At large gauge coupling it goes as $\lambda^{-1/2}$. The Anderson-Higgs mass increases with increasing λ and goes as λ^2 in the rashbon-BEC.
- A key outcome of this work is that we show that the rashbon-BEC can be described as a collection of anisotropically dispersing rashbons interacting via a contact interaction. We obtain an analytical expression for the rashbon-rashbon interaction for the spherical gauge field showing that it goes as λ^{-1} . We argue that this result is true for a generic gauge field (spin-orbit interaction).

We conclude the paper by revisiting the RG flow diagram of the two body problem introduced in ref. [16]. Fig. 4 is a schematic RG flow diagram in the λ - v plane for the two-particle problem. The key point is that flow from any point with $\lambda \neq 0$ and $v \neq 0$ reaches R which is the stable rashbon fixed point corresponding to $\lambda = \infty$ and

$v = -1$. Indeed, the properties of the state attained by a finite density of fermions at large λ is controlled by the rashbon fixed point; it is therefore a weakly interacting gas of rashbons – the rashbon BEC.

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